1. *Something Strange Is Afoot*

Quantum. The word is at once evocative, bewildering and fascinating. Depending on your point of view, it is either a testament to the profound success of science or a symbol of the limited scope of human intuition as we struggle with the inescapable strangeness of the subatomic domain. To a physicist, quantum mechanics is one of the three great pillars supporting our understanding of the natural world, the others being Einstein's theories of Special and General Relativity. Einstein's theories deal with the nature of space and time and the force of gravity. Quantum mechanics deals with everything else, and one can argue that it doesn't matter a jot whether it is evocative, bewildering or fascinating; it's simply a physical theory that describes the way things behave. Measured by this pragmatic yardstick, it is quite dazzling in its precision and explanatory power. There is a test of quantum electrodynamics, the oldest and most well understood of the modern quantum theories, which involves measuring the way an electron behaves in the vicinity of a magnet. Theoretical physicists worked hard for years using pens, paper and computers to predict what the experiments should find. Experimenters built and operated delicate experiments to tease out the finer details of Nature. Both camps independently returned precision results, comparable in their accuracy to measuring the distance between Manchester and New York to within a few centimetres. Remarkably, the number returned by the experimenters agreed exquisitely with that computed by the theorists; measurement and calculation were in perfect agreement.

This is impressive, but it is also esoteric, and if mapping the miniature were the only concern of quantum theory, you might be forgiven for wondering what all the fuss is about. Science, of course, has no brief to be useful, but many of the technological and social changes that have revolutionized our lives have arisen out of fundamental research carried out by modern-day explorers whose only motivation is to better understand the world around them. These curiosity-led voyages of discovery across all scientific disciplines have delivered increased life expectancy, intercontinental air travel, modern telecommunications, freedom from the drudgery of subsistence farming and a sweeping, inspiring and humbling vision of our place within an infinite sea of stars. But these are all in a sense spin-offs. We explore because we are curious, not because we wish to develop grand views of reality or better widgets.

Quantum theory is perhaps the prime example of the infinitely esoteric becoming the profoundly useful. Esoteric, because it describes a world in which a particle really can be in several places at once and moves from one place to another by exploring the entire Universe simultaneously. Useful, because understanding the behaviour of the smallest building blocks of the Universe underpins our understanding of everything else. This claim borders on the hubristic, because the world is filled with diverse and complex phenomena. Notwithstanding this complexity, we have discovered that everything is constructed out of a handful of tiny particles that move around according to the rules of quantum theory. The rules are so simple that they can be summarized on the back of an envelope. And the fact that we do not need a whole library of books to explain the essential nature of things is one of the greatest mysteries of all.

It appears that the more we understand about the elemental nature of the world, the simpler it looks. We will, in due course, explain what these basic rules are and how the tiny building blocks conspire to form the world. But, lest we get too dazzled by the underlying simplicity of the Universe, a word of caution is in order: although the basic rules of the game are simple, their consequences are not necessarily easy to calculate. Our everyday experience of the world is dominated by the relationships between vast collections of many trillions of atoms, and to try to derive the behaviour of plants and people from first principles would be folly. Admitting this does not diminish the point – all phenomena really are underpinned by the quantum physics of tiny particles.

Consider the world around you. You are holding a book made of paper, the crushed pulp of a tree.[1] Trees are machines able to take a supply of atoms and molecules, break them down and rearrange them into cooperating colonies composed of many trillions of individual parts. They do this using a molecule known as chlorophyll, composed of over a hundred carbon, hydrogen and oxygen atoms twisted into an intricate shape with a few magnesium and nitrogen atoms bolted on. This assembly of particles is able to capture the light that has travelled the 93 million miles from our star, a nuclear furnace the volume of a million earths, and transfer that energy into the heart of cells, where it is used to build molecules from carbon dioxide and water, giving out life-enriching oxygen as it does so. It's these molecular chains that form the superstructure of trees and all living things, and the paper in your book. You can read the book and understand the words because you have eyes that can convert the scattered light from the pages into electrical impulses that are interpreted by your brain, the most complex structure we know of in the Universe. We have discovered that all these things are nothing more than assemblies of atoms, and that the wide variety of atoms are constructed using only three particles: electrons, protons and neutrons. We have also discovered that the protons and neutrons are themselves made up of smaller entities called quarks, and that is where things stop, as far as we can tell today. Underpinning all of this is quantum theory.

The picture of the Universe we inhabit, as revealed by modern physics, is therefore one of underlying simplicity; elegant phenomena dance away out of sight and the diversity of the macroscopic world emerges. This is perhaps the crowning achievement of modern science; the reduction of the tremendous complexity in the world, human beings included, to a description of the behaviour of just

1. Unless of course you are reading an electronic version of the book, in which case you will need to exercise your imagination.

a handful of tiny subatomic particles and the four forces that act between them. The best descriptions we have of three of the forces, the strong and weak nuclear forces that operate deep within the atomic nucleus and the electromagnetic force that glues atoms and molecules together, are provided by quantum theory. Only gravity, the weakest but perhaps most familiar of the four, does not at present have a satisfactory quantum description.

Quantum theory does, admittedly, have something of a reputation for weirdness, and there have been reams of drivel penned in its name. Cats can be both alive and dead; particles can be in two places at once; Heisenberg says everything is uncertain. These things are all true, but the conclusion so often drawn – that since something strange is afoot in the microworld, we are steeped in mystery – is most definitely not. Extrasensory perception, mystical healing, vibrating bracelets to protect us from radiation and who-knows-what-else are regularly smuggled into the pantheon of the possible under the cover of the word 'quantum'. This is nonsense born from a lack of clarity of thought, wishful thinking, genuine or mischievous misunderstanding, or some unfortunate combination of all of the above. Quantum theory describes the world with precision, using mathematical laws as concrete as anything proposed by Newton or Galileo. That's why we can compute the magnetic response of an electron with such exquisite accuracy. Quantum theory provides a description of Nature that, as we shall discover, has immense predictive and explanatory power, spanning a vast range of phenomena from silicon chips to stars.

Our goal in writing this book is to demystify quantum theory; a theoretical framework that has proved famously confusing, even to its early practitioners. Our approach will be to adopt a modern perspective, with the benefit of a century of hindsight and theoretical developments. To set the scene, however, we would like to begin our journey at the turn of the twentieth century, and survey some of the problems that led physicists to take such a radical departure from what had gone before.

Quantum theory was precipitated, as is often the case in science, by the discovery of natural phenomena that could not be explained by the scientific paradigms of the time. For quantum theory these were many and varied. A cascade of inexplicable results caused excitement and confusion, and catalysed a period of experimental and theoretical innovation that truly deserves to be accorded that most clichéd label: a golden age. The names of the protagonists are etched into the consciousness of every student of physics and dominate undergraduate lecture courses even today: Rutherford, Bohr, Planck, Einstein, Pauli, Heisenberg, Schrödinger, Dirac. There will probably never again be a time in history where so many names become associated with scientific greatness in the pursuit of a single goal; a new theory of the atoms and forces that make up the physical world. In 1924, looking back on the early decades of quantum theory, Ernest Rutherford, the New-Zealand-born physicist who discovered the atomic nucleus in Manchester, wrote: 'The year 1896 . . . marked the beginning of what has been aptly termed the heroic age of Physical Science. Never before in the history of physics has there been witnessed such a period of intense activity when discoveries of fundamental importance have followed one another with such bewildering rapidity.'

But before we travel to nineteenth-century Paris and the birth of quantum theory, what of the word 'quantum' itself? The term entered physics in 1900, through the work of Max Planck. Planck was concerned with finding a theoretical description of the radiation emitted by hot objects – the so-called 'black body radiation' – apparently because he was commissioned to do so by an electric lighting company: the doors to the Universe have occasionally been opened by the prosaic. We will discuss Planck's great insight in more detail later in the book but, for the purposes of this brief introduction, suffice to say he found that he could only explain the properties of black body radiation if he assumed that light is emitted in little packets of energy, which he called 'quanta'. The word itself means 'packets' or 'discrete'. Initially, he thought that this was purely a mathematical trick, but subsequent work in 1905 by Albert Einstein on a phenomenon called the photoelectric effect gave further support to the quantum hypothesis. These results were suggestive, because little packets of energy might be taken to be synonymous with particles.

The idea that light consists of a stream of little bullets had a long and illustrious history dating back to the birth of modern physics and Isaac Newton. But Scottish physicist James Clerk Maxwell appeared to have comprehensively banished any lingering doubts in 1864 in a series of papers that Albert Einstein later described as 'the most profound and the most fruitful that physics has experienced since the time of Newton'. Maxwell showed that light is an electromagnetic wave, surging through space, so the idea of light as a wave had an immaculate and, it seemed, unimpeachable pedigree. Yet, in a series of experiments from 1923 to 1925 conducted at Washington University in Saint Louis, Arthur Compton and his co-workers succeeded in bouncing the quanta of light off electrons. Both behaved rather like billiard balls, providing clear evidence that Planck's theoretical conjecture had a firm grounding in the real world. In 1926, the light quanta were christened 'photons'. The evidence was incontrovertible – light behaves both as a wave and as a particle. That signalled the end for classical physics, and the end of the beginning for quantum theory.

2. *Being in Two Places at Once*

Ernest Rutherford cited 1896 as the beginning of the quantum revolution because this was the year Henri Becquerel, working in his laboratory in Paris, discovered radioactivity. Becquerel was attempting to use uranium compounds to generate X-rays, discovered just a few months previously by Wilhelm Röntgen in Würzburg. Instead, he found that uranium compounds emit 'les rayons uraniques', which were able to darken photographic plates even when they were wrapped in thick paper that no light could penetrate. The importance of Becquerel's rays was recognized in a review article by the great scientist Henri Poincaré as early as 1897, in which he wrote presciently of the discovery 'one can think today that it will open for us an access to a new world which no one suspected'. The puzzling thing about radioactive decay, which proved to be a hint of things to come, was that nothing seemed to trigger the emission of the rays; they just popped out of substances spontaneously and unpredictably.

In 1900, Rutherford noted the problem: 'all atoms formed at the same time should last for a definite interval. This, however, is contrary to the observed law of transformation, in which the atoms have a life embracing all values from zero to infinity.' This randomness in the behaviour of the microworld came as a shock because, until this point, science was resolutely deterministic. If, at some instant in time, you knew everything it is possible to know about something, then it was believed you could predict with absolute certainty what would happen to it in the future. The breakdown of this kind of predictability is a key feature of quantum theory: it deals with probabilities rather than certainties, not because we lack absolute knowledge, but because some aspects of Nature are, at their very heart, governed by the laws of chance. And so we now understand that it is simply impossible to predict when a particular atom will decay. Radioactive decay was science's first encounter with Nature's dice, and it confused many physicists for a long time.

Clearly, there was something interesting going on inside atoms, although their internal structure was entirely unknown. The key discovery was made by Rutherford in 1911, using a radioactive source to bombard a very thin sheet of gold with a type of radiation known as alpha particles (we now know them to be the nuclei of helium atoms). Rutherford, with his co-workers Hans Geiger and Ernest Marsden, discovered to their immense surprise that around 1 in 8,000 alpha particles did not fly through the gold as expected, but bounced straight back. Rutherford later described the moment in characteristically colourful language: 'It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.' By all accounts, Rutherford was an engaging and no-nonsense individual: he once described a self-important official as being 'like a Euclidean point: he has position without magnitude'.

Rutherford calculated that his experimental results could be explained only if the atom consists of a very small nucleus at the centre with electrons orbiting around it. At the time, he probably had in mind a situation similar to the planets orbiting around the Sun. The nucleus contains almost all the mass of the atom, which is why it is capable of stopping his '15-inch shell' alpha particles and bouncing them back. Hydrogen, the simplest element, has a nucleus consisting of a single proton with a radius of around 1.75×10^{-15} m. If you are unfamiliar with this notation, this means 0.00000000000000175 metres, or in words, just under two thousand million millionths of a metre. As far as we can tell today, the single electron is like Rutherford's self-important official, point-like, and it orbits around the hydrogen nucleus at a radius around 100,000 times larger than the nuclear diameter. The nucleus has a positive electric charge and the electron has a negative electric charge, which means there is an attractive force between them analogous to the force of gravity that holds the Earth in orbit around the Sun. This in turn means that atoms are largely empty space. If you imagine a nucleus scaled up to the size of a tennis ball, then the tiny electron would be smaller than a mote of dust orbiting at a distance of a kilometre. These figures are quite surprising because solid matter certainly does not feel very empty.

Rutherford's nuclear atom raised a host of problems for the physicists of the day. It was well known, for instance, that the electron should lose energy as it moves in orbit around the atomic nucleus, because all electrically charged things radiate energy away if they move in curved paths. This is the idea behind the operation of the radio transmitter, inside which electrons are made to jiggle and, as a result, electromagnetic radio waves issue forth. Heinrich Hertz invented the radio transmitter in 1887, and by the time Rutherford discovered the atomic nucleus there was a commercial radio station sending messages across the Atlantic from Ireland to Canada. So there was clearly nothing wrong with the theory of orbiting charges and the emission of radio waves, and that meant confusion for those trying to explain how electrons can stay in orbit around nuclei.

A similarly inexplicable phenomenon was the mystery of the light emitted by atoms when they are heated. As far back as 1853, the Swedish scientist Anders Jonas Ångstrom discharged a spark through a tube of hydrogen gas and analysed the emitted light. One might assume that a glowing gas would produce all the colours of the rainbow; after all, what is the Sun but a glowing ball of gas? Instead, Ångstrom observed that hydrogen emits light of three very distinct colours: red, blue-green and violet, like a rainbow with three pure, narrow arcs. It was soon discovered that each of the chemical elements behaves in this way, emitting a unique barcode of colours. By the time Rutherford's nuclear atom came along, a scientist named Heinrich Gustav Johannes Kayser had compiled a six-volume, 5,000-page reference work entitled *Handbuch der Spectroscopie*, documenting all the shining coloured lines from the known elements. The question, of course, was why? Not only 'why, Professor Kayser?' (he must have been great fun at dinner parties), but also 'why the profusion of coloured lines?' For over sixty years the science of

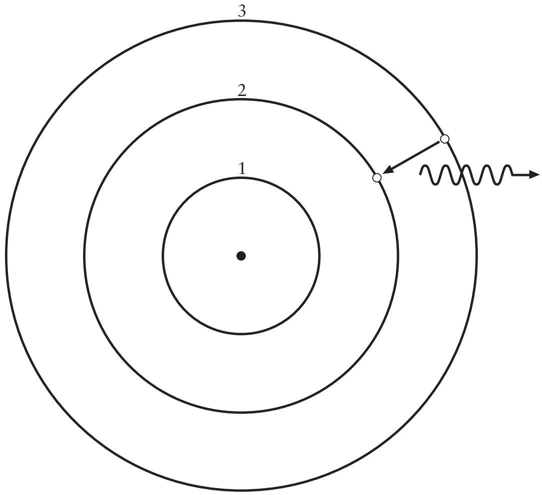

Figure 2.1: Bohr's model of an atom, illustrating the emission of a photon (wavy line) as an electron drops down from one orbit to another (indicated by the arrow).

spectroscopy, as it was known, had been simultaneously an observational triumph and a theoretical wasteland.

In March 1912, fascinated by the problem of atomic structure, Danish physicist Niels Bohr travelled to Manchester to meet with Rutherford. He later remarked that trying to decode the inner workings of the atom from the spectroscopic data had been akin to deriving the foundations of biology from the coloured wing of a butterfly. Rutherford's solar system atom provided the clue Bohr needed, and by 1913 he had published the first quantum theory of atomic structure. The theory certainly had its problems, but it did contain several key insights that triggered the development of modern quantum theory. Bohr concluded that electrons can only take up certain orbits around the nucleus with the lowest-energy orbit lying closest in. He also said that electrons are able to jump between these orbits. They jump out to a higher orbit when they receive energy (from a spark in a tube for example) and, in time, they will fall back down, emitting light in the process. The colour of the light is determined directly by the energy difference between the two orbits. Figure 2.1 illustrates the basic idea; the arrow represents an electron as it jumps from the third energy level down to the second energy level, emitting light (represented by the wavy line) as it does so. In Bohr's model, the electron is only allowed to orbit the proton in one of these special, 'quantized', orbits; spiralling inwards is simply forbidden. In this way Bohr's model allowed him to compute the wavelengths (i.e. colours) of light observed by Ångstrom – they were to be attributed to an electron hopping from the fifth orbit down to the second orbit (the violet light), from the fourth orbit down to the second (the blue-green light) or from the third orbit down to the second (the red light). Bohr's model also correctly predicted that there should be light emitted as a result of electrons hopping down to the first orbit. This light is in the ultra-violet part of the spectrum, which is not visible to the human eye, and so it was not seen by Ångstrom. It had, however, been spotted in 1906 by Harvard physicist Theodore Lyman, and Bohr's model described Lyman's data beautifully.

Although Bohr did not manage to extend his model beyond hydrogen, the ideas he introduced could be applied to other atoms. In particular, if one supposes that the atoms of each element have a unique set of orbits then they will only ever emit light of certain colours. The colours emitted by an atom therefore act like a fingerprint, and astronomers were certainly not slow to exploit the uniqueness of the spectral lines emitted by atoms as a way to determine the chemical composition of the stars.

Bohr's model was a good start, but it was clearly unsatisfactory: just why were electrons forbidden from spiralling inwards when it was known that they should lose energy by emitting electromagnetic waves – an idea so firmly rooted in reality with the advent of radio? And why are the electron orbits quantized in the first place?

And what about the heavier elements beyond hydrogen: how was one to go about understanding their structure?

Half-baked though Bohr's theory may have been, it was a crucial step, and an example of how scientists often make progress. There is no point at all in getting completely stuck in the face of perplexing and often quite baffling evidence. In such cases, scientists often make an ansatz, an educated guess if you like, and then proceed to compute the consequences of the guess. If the guess works, in the sense that the subsequent theory agrees with experiment, then you can go back with some confidence to try to understand your initial guess in more detail. Bohr's ansatz remained successful but unexplained for thirteen years.

We will revisit the history of these early quantum ideas as the book unfolds, but for now we leave a mass of strange results and half-answered questions, because this is what the early founders of quantum theory were faced with. In summary, following Planck, Einstein introduced the idea that light is made up of particles, but Maxwell had shown that light also behaves like waves. Rutherford and Bohr led the way in understanding the structure of atoms, but the way that electrons behave inside atoms was not in accord with any known theory. And the diverse phenomena collectively known as radioactivity, in which atoms spontaneously split apart for no discernible reason, remained a mystery, not least because it introduced a disturbingly random element into physics. There was no doubt about it: something strange was afoot in the subatomic world.

The first step towards a consistent, unified answer is widely credited to the German physicist Werner Heisenberg, and what he did represented nothing less than a completely new approach to the theory of matter and forces. In July of 1925, Heisenberg published a paper throwing out the old hotchpotch of ideas and half-theories, including Bohr's model of the atom, and ushered in an entirely new approach to physics. He began: 'In this paper it will be attempted to secure the foundations for a quantum theoretical mechanics which is exclusively based on relations between quantities which in principle are observable.' This is an important step, because Heisenberg is saying that the underlying mathematics of quantum theory need not correspond to anything with which we are familiar. The job of quantum theory should be to predict directly observable things, such as the colour of the light emitted from hydrogen atoms. It should not be expected to provide some kind of satisfying mental picture for the internal workings of the atom, because this is not necessary and it may not even be possible. In one fell swoop, Heisenberg removed the conceit that the workings of Nature should necessarily accord with common sense. This is not to say that a theory of the subatomic world shouldn't be expected to accord with our everyday experience when it comes to describing the motion of large objects, like tennis balls and aircraft. But we should be prepared to abandon the prejudice that small things behave like smaller versions of big things, if this is what our experimental observations dictate.

There is no doubt that quantum theory is tricky, and absolutely no doubt that Heisenberg's approach is extremely tricky indeed. Nobel Laureate Steven Weinberg, one of the greatest living physicists, wrote of Heisenberg's 1925 paper:

> If the reader is mystified at what Heisenberg was doing, he or she is not alone. I have tried several times to read the paper that Heisenberg wrote on returning from Heligoland, and, although I think I understand quantum mechanics, I have never understood Heisenberg's motivations for the mathematical steps in his paper. Theoretical physicists in their most successful work tend to play one of two roles: they are either sages or magicians . . . It is usually not difficult to understand the papers of sage-physicists, but the papers of magician-physicists are often incomprehensible. In that sense, Heisenberg's 1925 paper was pure magic.

Heisenberg's philosophy, though, is not pure magic. It is simple and it lies at the heart of our approach in this book: the job of a theory of Nature is to make predictions for quantities that can be compared to experimental results. We are not mandated to produce a theory that bears any relation to the way we perceive the world at large. Fortunately, although we are adopting Heisenberg's philosophy, we shall be following Richard Feynman's more transparent approach to the quantum world.

We've used the word 'theory' liberally in the last few pages and, before we continue to build quantum theory, it will be useful to take a look at a simpler theory in more detail. A good scientific theory specifies a set of rules that determine what can and cannot happen to some portion of the world. They must allow predictions to be made that can be tested by observation. If the predictions are shown to be false, the theory is wrong and must be replaced. If the predictions are in accord with observation, the theory survives. No theory is 'true' in the sense that it must always be possible to falsify it. As the biologist Thomas Huxley wrote: 'Science is organized common sense where many a beautiful theory was killed by an ugly fact.' Any theory that is not amenable to falsification is not a scientific theory – indeed one might go as far as to say that it has no reliable information content at all. The reliance on falsification is why scientific theories are different from matters of opinion. This scientific meaning of the word 'theory', by the way, is different from its ordinary usage, where it often suggests a degree of speculation. Scientific theories may be speculative if they have not yet been confronted with the evidence, but an established theory is something that is supported by a large body of evidence. Scientists strive to develop theories that encompass as wide a range of phenomena as possible, and physicists in particular tend to get very excited about the prospect of describing everything that can happen in the material world in terms of a small number of rules.

One example of a good theory that has a wide range of applicability is Isaac Newton's theory of gravity, published on 5 July 1687 in his *Philosophiæ Naturalis Principia Mathematica*. It was the first modern scientific theory, and although it has subsequently been shown to be inaccurate in some circumstances, it was so good that it is still used today. Einstein developed a more precise theory of gravity, General Relativity, in 1915.

Newton's description of gravity can be captured in a single mathematical equation:

$$F = G\frac{m_1 m_2}{r^2}$$

This may look simple or complicated, depending on your mathematical background. We do occasionally make use of mathematics as this book unfolds. For those readers who find the maths difficult, our advice is to skip over the equations without worrying too much. We will always try to emphasize the key ideas in a way that does not rely on the maths. The maths is included mainly because it allows us to really explain why things are the way they are. Without it, we should have to resort to the physicist-guru mentality whereby we pluck profundities out of thin air, and neither author would be comfortable with guru status.

Now let us return to Newton's equation. Imagine there is an apple hanging precariously from a branch. The consideration of the force of gravity triggered by a particularly ripe apple bouncing off his head one summer's afternoon was, according to folklore, Newton's route to his theory. Newton said that the apple is subject to the force of gravity, which pulls it towards the ground, and that force is represented in the equation by the symbol F. So, first of all, the equation allows you to calculate the force on the apple if you know what the symbols on the right-hand side of the equals sign mean. The symbol r stands for the distance between the centre of the apple and the centre of the Earth. It's r^2 because Newton discovered that the force depends on the square of the distance between the objects. In non-mathematical language, this means that if you double the distance between the apple and the centre of the Earth, the gravitational force drops by a factor of 4. If you triple the distance, it drops by a factor of 9. And so on. Physicists call this behaviour an inverse square law. The symbols m_1 and m_2 stand for the mass of the apple and the mass of the Earth, and their appearance encodes Newton's recognition that the gravitational force of attraction between two objects depends on the product of their masses. That then begs the question: what is mass? This is an interesting question in itself, and for the deepest answer available today we'll need to wait until we talk about a quantum particle known as the Higgs boson. Roughly speaking, mass is a measure of the amount of 'stuff' in something; the Earth is more massive than the apple. This kind of statement isn't really good enough, though. Fortunately Newton also provided a way of measuring the mass of an object independently of his law of gravitation, and it is encapsulated in the second of his three laws of motion, the ones so beloved of every high school student of physics:

1. Every object remains in a state of rest or uniform motion in a straight line unless it is acted upon by a force;
2. An object of mass m undergoes an acceleration a when acted upon by a force F. In the form of an equation, this reads $F = ma$;
3. To every action there is an equal and opposite reaction.

Newton's three laws provide a framework for describing the motion of things under the influence of a force. The first law describes what happens to an object when no forces act: the object either just sits still or moves in a straight line at constant speed. We shall be looking for an equivalent statement for quantum particles later on, and it's not giving the game away too much to say that quantum particles do not just sit still – they leap around all over the place even when no forces are present. In fact, the very notion of 'force' is absent in the quantum theory, and so Newton's second law is bound for the wastepaper basket too. We do mean that, by the way – Newton's laws are heading for the bin because they have been exposed as only approximately correct. They work well in many instances but fail totally when it comes to describing quantum phenomena. The laws of quantum theory replace Newton's laws and furnish a more accurate description of the world. Newton's physics emerges out of the quantum description, and it is important to realize that the situation is not 'Newton for big things and quantum for small': it is quantum all the way.