

# **NUMBERS GUIDE**

The Essentials of Business Numeracy

Sixth edition

# THE ECONOMIST IN ASSOCIATION WITH PROFILE BOOKS LTD

Published by Profile Books Ltd 3A Exmouth House, Pine Street, London EC1R oJH www.profilebooks.com

First published by The Economist Books Ltd 1991

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Typeset by International Typesetters Inc. *info@InternationalTypesetters.com* 

Printed in Great Britain by Creative Print and Design (Wales), Ebbw Vale

A CIP catalogue record for this book is available from the British Library

Hardback 978 1 84668 171 4 Paperback 978 1 84668 903 1 E-book 978 1 84765 934 7

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### Introduction

"Statistical thinking will one day be as necessary a qualification for efficient citizenship as the ability to read and write."

H.G. Wells

THIS BOOK is about solving problems and making decisions using numerical methods. Everyone – people in business, social administrators, bankers – can do their jobs better if equipped with such tools. No special skills or prior knowledge are required. Numerical methods amount to little more than applied logic: they all reduce to step-by-step instructions and can be processed by simple computing devices. Yet numerical methods are exciting and powerful. They work magic, which is perhaps why they are shrouded in mystery. This book strips away that mystery and provides a guided tour through the statistical workshop. There are no secrets, no barriers to entry. Anyone can use these tools. Everyone should.

#### What are numerical methods?

Numerical methods range from the simple (how to calculate percentages and interest) to the relatively complex (how to evaluate competing investment opportunities); from the concrete (how to find the shortest route for deliveries) to the vague (how to deal with possible levels of sales or market share). The link is quantitative analysis, a scientific approach.

This does not mean that qualitative factors (intangibles such as personal opinion, hunch, technological change and environmental awareness) should be ignored. On the contrary, they must be brought into the decision process, but in a clear, unemotional way. Thus, a major part of this book is devoted to dealing with risk. After all, this forms a major part of the business environment. Quantifying risk and incorporating it into the decision-making process is essential for successful business.

In bringing together quantitative techniques, the book borrows heavily from mathematics and statistics and also from other fields, such as accounting and economics.

#### A brief summary

We all perform better when we understand why we are doing something. For this reason, this book always attempts to explain why as well as how methods work. Proofs are not given in the rigorous mathematical sense, but the techniques are explained in such a way that the reader should be able to gain at least an intuitive understanding of why they work. This should also aid students who use this book as an introduction to heavier statistical or mathematical works.

The techniques are illustrated with business examples where possible but sometimes abstract illustrations are preferred. This is particularly true of probability, which is useful for assessing business risk but easier to understand through gamblers' playing cards and coins.

Examples use many different currencies and both metric and imperial measurements. The SI standards for measurement (see SI units in the A–Z) are excellent, but they are generally ignored here in favour of notation and units which may be more familiar to a wider audience.

This book works from the general to the particular.

**Chapter 1** lays the groundwork by running over some key concepts. Items of particular interest include proportions and percentages (which appear in many problems) and probability (which forms a basis for assessing risk).

**Chapter 2** examines ways of dealing with problems and decisions involving money, as many or most do. Interest, inflation and exchange rates are all covered. Note that the proportions met in the previous chapter are used as a basis for calculating interest and evaluating investment projects.

**Chapter 3** looks at summary measures (such as averages) which are important tools for interpretation and analysis. In particular, they unlock what is called the normal distribution, which is invaluable for modelling risk.

**Chapter 4** reviews the way data are ordered and interpreted using charts and tables. A series of illustrations draws attention to the benefits and shortfalls of various types of presentation.

**Chapter 5** examines the vast topic of forecasting. Few jobs can be done successfully without peering into the future. The objective is to pull together a view of the future in order to enhance the inputs to decision-making.

**Chapter 6** marks a turning point. It starts by considering the way that sampling saves time and money when collecting the inputs to decisions. This is a continuation of the theme in the previous chapters. However,

the chapter then goes on to look at ways of reaching the best decision from sample data. The techniques are important for better decisionmaking in general.

**Chapter 7** expands on the decision theme. It combines judgment with the rigour of numerical methods for better decisions in those cases which involve uncertainty and risk.

**Chapter 8** looks at some rather exciting applications of techniques already discussed. It covers:

- game strategy (for decision-making in competitive situations);
- queueing (for dealing with a wide range of business problems, only one of which involves customers waiting in line);
- stock control (critical for minimising costs);
- Markov chains (for handling situations where events in the future are directly affected by preceding events);
- roject management (with particular attention to risk); and
- simulation (for trying out business ideas without risking humiliation or loss).

**Chapter 9** reviews powerful methods for reaching the best possible decision when risk is not a key factor.

**An A-Z section** concludes the book. It gives key definitions. It covers a few terms which do not have a place in the main body of the book. And it provides some useful reference material, such as conversion factors and formulae for calculating areas and volumes.

Additional information is available on this book's website at www.NumbersGuide.com.

#### How to use this book

There are four main approaches to using this book:

- 1 If you want to know the meaning of a mathematical or statistical term, consult the A–Z. If you want further information, turn to the cross-reference in the A–Z entry, shown in small capital letters, and read more.
- 2 If you want to know about a particular numerical method, turn to the appropriate chapter and read all about it.
- 3 If you have a business problem that needs solving, use the A-Z, the contents page, or this chapter for guidance on the methods available, then delve deeper.

4 If you are familiar with what to do but have forgotten the detail, then formulae and other reference material are highlighted throughout the book.

#### Calculators and computers

There can be few people who are not familiar with electronic calculators. If you are selecting a new calculator for use on your computer or hand-held device, choose one with basic operations ( $+ - \times$  and  $\div$ ) and at least one memory, together with the following:

- Exponents and roots (probably summoned by keys marked x<sup>y</sup> and x<sup>1/y</sup>): essential for dealing with growth rates, compound interest and inflation.
- Factorials (look for a key such as x!): useful for calculating permutations and combinations.
- Logarithms (log and 10<sup>x</sup> or ln and e<sup>x</sup>): less important but sometimes useful.
- Trigonometric functions (sin, cos and tan): again, not essential, but handy for some calculations (see Triangles and trigonometry in the A-Z).
- Base/radix conversion/operations: useful for exploring bases, and for doing boffinish stuff with computers.
- Constants  $\pi$  and e: occasionally useful.
- Net present value and internal rate of return (NPV and IRR). These are found only on financial calculators. They assist investment evaluation, but you will probably prefer to use a spreadsheet.

Note that the calculator in Microsoft Windows has most of the above functions when used in scientific mode.

Computer and maybe tablet users will find themselves turning to a spreadsheet program to try out many of the techniques in this book. Spreadsheets take the tedium out of many operations and are more or less essential for some activities such as simulation.

For the non-initiated, a spreadsheet is like a huge sheet of blank paper divided up into little boxes (known as cells). You can key text, numbers or instructions into any of the cells. If you enter ten different values in a sequence of ten cells, you can then enter an instruction in the eleventh, perhaps telling the computer to add or multiply the ten values together. One powerful feature is that you can copy things from one cell to another almost effortlessly. Tedious, repetitive tasks become simple. Another handy feature is the large selection of instructions (or functions) which enable you to do much more complex things than you would with a calculator. Lastly, spreadsheets also produce charts which are handy for interpretation and review.

The market leader in spreadsheet programs is Microsoft Excel (packaged with Microsoft Office). It is on the majority of corporate desktops. Good alternatives to MS Office, which happen to be free and include spreadsheets broadly compatible with Excel, include OpenOffice and Google Docs.

#### Conclusion

There are so many numerical methods and potential business problems that it is impossible to cross-reference them all. Use this book to locate techniques applicable to your problems and take the following steps:

- Define the problem clearly.
- Identify the appropriate technique.
- Collect the necessary data.
- Develop a solution.
- Analyse the results.
- Start again if necessary or implement the results.

The development of sophisticated computer packages has made it easy for anyone to run regressions, to identify relationships or to make forecasts. But averages and trends often conceal more than they reveal. Never accept results out of hand. Always question whether your analysis may have led you to a faulty solution. For example, you may be correct in noting a link between national alcohol consumption and business failures; but is one directly related to the other, or are they both linked to some unidentified third factor?

### 1 Key concepts

"Round numbers are always false."

Samuel Johnson

#### Summary

Handling numbers is not difficult. However, it is important to be clear about the basics. Get them right and everything else slots neatly into place.

People tend to be comfortable with percentages, but it is easy to perform many calculations using proportions. The two are used interchangeably throughout this book. When a result is obtained as a proportion, such as  $6 \div 100 = 0.06$ , this is often referred to as 6%. Sums become much easier when you can convert between percentages and proportions by eye: just shift the decimal point two places along the line (adding zeros if necessary).

Proportions simplify problems involving growth, reflecting perhaps changes in demand, interest rates or inflation. Compounding by multiplying by one plus a proportion several times (raising to a power) is the key. For example, compound growth of 6% per year over two years increases a sum of money by a factor of  $1.06 \times 1.06 = 1.1236$ . So \$100 growing at 6% per year for two years increases to  $100 \times 1.1236 = 112.36$ .

Proportions are also used in probability, which is worth looking at for its help in assessing risks.

Lastly, index numbers are introduced in this chapter.

#### Ways of looking at data

It is useful to be aware of different ways of categorising information. This is relevant for three main reasons.

#### 1 Time series and cross-sectional data

Certain problems are found with some types of data only. For example, it is not too hard to see that you would look for seasonal and cyclical trends in time series but not in cross-sectional data.

Time series record developments over time; for example, monthly

ice cream output, or a ten-year run of the finance director's annual salary.

**Cross-sectional data** are snapshots that capture a situation at a moment in time, such as the value of sales at various branches on one day.

#### 2 Scales of measurement

Some techniques are used with one type of data only. A few of the sampling methods in Chapter 7 are used only with data which are measured on an interval or ratio scale. Other sampling methods apply to nominal or ordinal scale information only.

Nominal or categorical data identify classifications only. No particular quantities are implied. Examples include sex (male/female), departments (international/marketing/personnel) and sales regions (area number 1, 2, 3, 4).

**Ordinal or ranked data.** Categories can be sorted into a meaningful order, but differences between ranks are not necessarily equal. What do you think of this politician (awful, satisfactory, wonderful)? What grade of wheat is this (A1, A2, B1...)?

**Interval scale data.** Measurable differences are identified, but the zero point is arbitrary. Is 20° Celsius twice as hot as 10°C? Convert to Fahrenheit to see that it is not. The equivalents are  $68^{\circ}$ F and  $50^{\circ}$ F. Temperature is measured on an interval scale with arbitrary zero points (0°C and  $32^{\circ}$ F).

**Ratio scale data.** There is a true zero and measurements can be compared as ratios. If three frogs weigh 250gm, 500gm and 1,000gm, it is clear that Mr Frog is twice as heavy as Mrs Frog, and four times the weight of the baby.

#### 3 Continuity

Some results are presented in one type of data only. You would not want to use a technique which tells you to send 0.4 of a salesman on an assignment, when there is an alternative technique which deals in whole numbers.

**Discrete values** are counted in whole numbers (integers): the number of frogs in a pond, the number of packets of Fat Cat Treats sold each week.

**Continuous variables** do not increase in steps. Measurements such as heights and weights are continuous. They can only be estimated: the temperature is 25°C; this frog weighs 500gm. The accuracy of such estimates depends on the precision of the measuring instrument. More

accurate scales might show the weight of the frog at 501 or 500.5 or 500.0005 gm, etc.

#### Fractions, percentages and proportions

#### Fractions

Fractions are not complicated. Most monetary systems are based on 100 subdivisions: 100 cents to the dollar or euro, or 100 centimes to the Swiss franc. Amounts less than one big unit are fractions; 50 cents is half, or 0.50, or 50% of one euro. Common (vulgar) fractions ( $\frac{1}{2}$ ), decimal fractions (0.50), proportions (0.50) and percentages (50%) are all the same thing with different names. Convert any common fraction to a decimal fraction by dividing the lower number (denominator) into the upper number (numerator). For example,  $\frac{3}{4} = 3 \div 4 = 0.75$ . The result is also known as a proportion. Multiply it by 100 to convert it into a percentage. Recognition of these simple relationships is vital for easy handling of numerical problems.

**Decimal places.** The digits to the right of a decimal point are known as decimal places. 1.11 has two decimal places, 1.111 has three, 1.1111 has four, and so on.

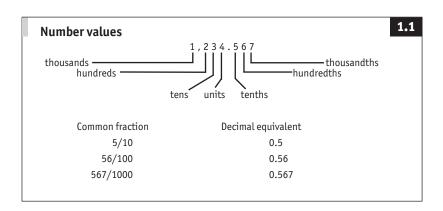
**Reading decimal fractions.** Reading \$10.45m as ten-point-forty-five million dollars will upset the company statistician. Decimal fractions are read out figure-by-figure: ten-point-four-five in this example. Forty-five implies four tens and five units, which is how it is to the left of

#### Percentage points and basis points

**Percentages** and percentage changes are sometimes confused. If an interest rate or inflation rate increases from 10% to 12%, it has risen by two units, or two percentage points. But the percentage increase is 20% (=  $2 \div 10 \times 100$ ). Take care to distinguish between the two.

**Basis points**. Financiers attempt to profit from very small changes in interest or exchange rates. For this reason, one unit, say 1% (ie, one percentage point) is often divided into 100 basis points:

- 1 basis point = 0.01 percentage point
- 10 basis points = 0.10 percentage point
- 25 basis points = 0.25 percentage point
- 100 basis points = 1.00 percentage point



#### Percentage increases and decreases

A percentage increase followed by the same percentage decrease does not leave you back where you started. It leaves you worse off. Do not accept a 50% increase in salary for six months, to be followed by a 50% cut.

- \$1,000 increased by 50% is \$1,500.
- 50% of \$1,500 is \$750.

A frequent business problem is finding what a number was before it was increased by a given percentage. Simply divide by (1 + i), where i is the percentage increase expressed as a proportion. For example:

I if an invoice is for €575 including 15% VAT (value added tax, a sales tax) the taxexclusive amount is €575  $\div$  1.15 = €500.

**Fractions.** If anything is increased by an amount  $x_y$ , the increment is  $x_y$  + y of the new total:

- if €100 is increased by 1/2, the increment of €50 is 1/(1 + 2) = 1/3 of the new total of €150;
- ▶ ¥100 increased by  $\frac{3}{4}$  is ¥175; the ¥75 increment is  $\frac{3}{3} + \frac{4}{3} = \frac{3}{7}$  of the new ¥175 total.

the decimal point. To the right, the fractional amounts shrink further to tenths, hundredths, and so on. (See Figure 1.1.)

**Think of two fractions.** It is interesting to reflect that fractions go on for ever. Think of one fractional amount; there is always a smaller one. Think of two fractions; no matter how close together they are, there is

#### How big is a billion?

As individuals we tend to deal with relatively small amounts of cash. As corporate people, we think in units of perhaps one million at a time. In government, money seemingly comes in units of one billion only.

**Scale**. The final column below, showing that a billion seconds is about 32 years, gives some idea of scale. The fact that Neanderthal man faded away a mere one trillion seconds ago is worth a thought.

| Quantity | Zeros | Scientific      | In numbers        | In seconds        |
|----------|-------|-----------------|-------------------|-------------------|
| Thousand | 3     | $1	imes10^3$    | 1,000             | 17 minutes        |
| Million  | 6     | $1	imes 10^6$   | 1,000,000         | 11 ½ days         |
| Billion  | 9     | $1	imes 10^9$   | 1,000,000,000     | 32 years          |
| Trillion | 12    | $1	imes10^{12}$ | 1,000,000,000,000 | 32 thousand years |

**British billions.** The number of zeros shown are those in common use. The British billion (with 12 rather than 9 zeros) is falling out of use. It is not used in this book.

**Scientific notation.** Scientific notation can be used to save time writing out large and small numbers. Just shift the decimal point along by the number of places indicated by the exponent (the little number in the air). For example:

- $1.25 \times 10^6$  is a shorthand way of writing 1,250,000;
- $\blacksquare~~1.25\times10^{-6}$  is the same as 0.00000125.

Some calculators display very large or very small answers this way. Test by keying 1  $\div$  501. The calculator's display might show 1.996 -03, which means 1.996  $\times$  10<sup>-3</sup> or 0.001996. You can sometimes convert such displays into meaningful numbers by adding 1. Many calculators showing 1.996 -03 would respond to you keying + 1 by showing 1.001996. This helps identify the correct location for the decimal point.

always another one to go in between. This brings us to the need for rounding.

#### Rounding

An amount such as \$99.99 is quoted to two decimal places when selling, but usually rounded to \$100 in the buyer's mind. The Japanese have stopped counting their sen. Otherwise they would need wider calculators. A few countries are perverse enough to have currencies with three places of decimal: 1,000 fils = 1 dinar. But 1 fil coins are generally no longer in use and values such as 1.503 are rounded off to 1.505. How do you round 1.225 if there are no 5 fil coins? It depends whether you are buying or selling.

Generally, aim for consistency when rounding. Most calculators and spreadsheets achieve this by adopting the 4/5 principle. Values ending in 4 or less are rounded down (1.24 becomes 1.2), amounts ending in 5 or more are rounded up (1.25 becomes 1.3). Occasionally this causes problems.

**Two times two equals four.** Wrong: the answer could be anywhere between two and six when dealing with rounded numbers.

- 1.5 and 2.4 both round to 2 (using the 4/5 rule)
- 1.5 multiplied by 1.5 is 2.25, which rounds to 2
- 2.4 multiplied by 2.4 is 5.76, which rounds to 6

Also note that 1.45 rounds to 1.5, which rounds a second time to 2, despite the original being nearer to 1.

The moral is that you should take care with rounding. Do it after multiplying or dividing. When starting with rounded numbers, never quote the answer to more significant figures (see below) than the least precise original value.

#### Significant figures

Significant figures convey precision. Take the report that certain American manufacturers produced 6,193,164 refrigerators in a particular year. For some purposes, it may be important to know that exact number. Often, though, 6.2m, or even 6m, conveys the message with enough precision and a good deal more clarity. The first value in this paragraph is quoted to seven significant figures. The same amount to two significant figures is 6.2m (or 6,200,000). Indeed, had the first amount been estimated from refrigerator-makers' turnover and the average sale price of a refrigerator, seven-figure approximation would be spurious accuracy, of which economists are frequently guilty.

**Significant figures and decimal places in use.** Three or four significant figures with up to two decimal places are usually adequate for discussion purposes, particularly with woolly economic data. (This is sometimes called three or four effective figures.) Avoid decimals where possible, but do not neglect precision when it is required. Bankers would cease to make a profit if they did not use all the decimal places on their calculators when converting exchange rates.

#### **Percentages and proportions**

Percentages and proportions are familiar through money: 45 cents is 45% of 100 cents, or, proportionately, 0.45 of one dollar. Proportions are expressed relative to one, percentages in relation to 100. Put another way, a percentage is a proportion multiplied by 100. This is a handy thing to know when using a calculator.

Suppose a widget which cost \$200 last year now retails for \$220. Proportionately, the current cost is 1.1 times the old price (220  $\div$  200 = 1.1). As a percentage, it is 110% of the original (1.1  $\times$  100 = 110).

In common jargon, the new price is 10% higher. The percentage increase (the 10% figure) can be found in any one of several ways. The most painless is usually to calculate the proportion ( $220 \div 200 = 1.1$ ); subtract 1 from the answer (1.10 - 1 = 0.10); and multiply by 100 ( $0.10 \times 100 = 10$ ). Try using a calculator for the division and doing the rest by eye; it's fast.

**Proportions and growth.** The relationship between proportions and percentages is astoundingly useful for compounding.

The finance director has received annual 10% pay rises for the last ten years. By how much has her salary increased? Not 100%, but nearly 160%. Think of the proportionate increase. Each year, she earned 1.1 times the amount in the year before. In year one she received the base amount (1.0) times 1.1 = 1.1. In year two, total growth was  $1.1 \times 1.1 = 1.21$ . In year three,  $1.21 \times 1.1 = 1.331$ , and so on up to  $2.358 \times 1.1 = 2.594$  in the tenth year. Take away 1 and multiply by 100 to reveal the 159.4 percentage increase over the whole period.

**Powers.** The short cut when the growth rate is always the same, is to recognise that the calculation involves multiplying the proportion by itself a number of times. In the previous example, 1.1 was multiplied by itself 10 times. In math-speak, this is called raising 1.1 to the power of 10 and is written 1.1<sup>10</sup>.

The same trick can be used to "annualise" monthly or quarterly rates of growth. For example, a monthly rise in prices of 2.0% is equivalent to an annual rate of inflation of 26.8%, not 24%. The statistical section in the back of The Economist each week shows for over 40 countries the annualised percentage changes in output in the latest quarter compared with the previous quarter. If America's GDP is 1.7% higher during the January-March quarter than during the October-December quarter, then this is equivalent to an annual rate of increase of 7% (1.017 imes 1.017 imes1.017 × 1.017).

Using a calculator. Good calculators have a key marked something like  $x^{y}$ , which means x (any number) raised to the power of y (any other number). Key 1.1  $x^y$  10 = and the answer 2.5937... pops up on the display. It is that easy. To go back in the other direction, use the  $x^{1/y}$  key. So 2.5937  $x^{1/y}$  10 = gives the answer 1.1. This tells you that the number that has to be multiplied by itself 10 times to give 2.5937 is 1.1. (See also Growth rates and exponents box in Chapter 2.)

#### Table 1.1 Mr and Mrs Average's shopping basket

|              | А        | В              | С              |  |
|--------------|----------|----------------|----------------|--|
|              | €        | Jan 2012 = 100 | Jan 2014 = 100 |  |
| January 2012 | 1,568.34 | 100.0          | 86.6           |  |
| January 2013 | 1,646.76 | 105.0          | 90.9           |  |
| January 2014 | 1,811.43 | 115.5          | 100.0          |  |

Each number in column B = number in column A divided by (1,568.34  $\div$  100). Each number in column C = number in column B divided by  $(1,811.43 \div 100)$ .

#### Table 1.2 A base-weighted index of living costs

|              | А     | В     | С     |  |
|--------------|-------|-------|-------|--|
|              | Food  | Other | Total |  |
| Weights:     | 0.20  | 0.80  | 1.00  |  |
| January 2012 | 100.0 | 100.0 | 100.0 |  |
| January 2013 | 103.0 | 105.5 | 105.0 |  |
| January 2014 | 108.0 | 117.4 | 115.5 |  |
|              |       |       |       |  |

Each monthly value in column  $C = (\text{column A} \times 0.20) + (\text{column B} \times 0.80).$ Eg, for January 2014 (108.0  $\times$  0.20) + (117.4  $\times$  0.80) = 115.5.

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#### Table 1.3 A current-weighted index of living costs

| A     | В   | С  | D  | E   |   |
|-------|---|--|--|---|---|
| Food  | Food  | Other  | Other  | Total   |   |
| index | weight  | index  | weight   |   |   |
| 100.0 | 0.80  | 100.0  | 0.20   | 100.0   |   |
| 103.0 | 0.70  | 105.5  | 0.30   | 103.8   |   |
| 108.0 | 0.60  | 117.4  | 0.40   | 111.8   |   |
|       | <i>Food</i><br><i>index</i><br>100.0<br>103.0 | Food Food   index weight   100.0 0.80   103.0 0.70 | Food Food Other   index weight index   100.0 0.80 100.0   103.0 0.70 105.5 | Food Food Other Other   index weight index weight   100.0 0.80 100.0 0.20   103.0 0.70 105.5 0.30 | Food Food Other Other Total   index weight index weight 100.0 100 |

Each value in column E is equal to (number in column A  $\times$  weight in column B) + (number in column C  $\times$  weight in column D).

Eg, for January 2014 (108.0  $\times$  0.60) + (117.4  $\times$  0.40) = 111.8

#### **Index numbers**

There comes a time when money is not enough, or too much, depending on how you look at it. For example, the consumer prices index (also known as the cost of living or retail prices index) attempts to measure inflation as experienced by Mr and Mrs Average. The concept is straightforward: value all the items in the Average household's monthly shopping basket; do the same at some later date; and see how the overall cost has changed. However, the monetary totals, say €1,568.34 and €1,646.76, are not easy to handle and they distract from the task in hand. A solution is to convert them into index numbers. Call the base value 100. Then calculate subsequent values based on the percentage change from the initial amount. The second shopping basket cost 5% more, so the second index value is 105. A further 10% rise would take the index to 115.5.

To convert any series of numbers to an index:

- choose a base value (eg,  $\in$ 1,568.34 in the example here);
- divide it by 100, which will preserve the correct number of decimal places; then
- ✓ divide every reading by this amount.

Table 1.1 shows how this is done in practice.

**Rebasing.** To rebase an index so that some new period equals 100, simply divide every number by the value of the new base (Table 1.1). **Composite indices and weighting.** Two or more sub-indices are often combined to form one composite index. Instead of one cost of living index for the Averages, there might be two: showing expenditure on food, and all other spending. How should they be combined?

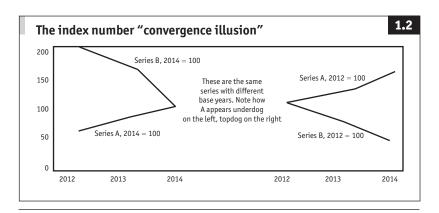


Table 1.4 Index comparisons

|                      | GDP         | Index    | Index           | Index         |
|----------------------|-------------|----------|-----------------|---------------|
|                      | per head \$ | US = 100 | <i>UK</i> = 100 | Germany = 100 |
| Monaco               | 171,465     | 356.4    | 439.9           | 389.5         |
| Luxembourg           | 114,232     | 237.4    | 293.1           | 259.5         |
| Norway               | 98,081      | 203.9    | 251.7           | 222.8         |
| Qatar                | 92,501      | 192.3    | 237.3           | 210.1         |
| Switzerland          | 83,326      | 173.2    | 213.8           | 189.3         |
| Macao SAR, China     | 65,550      | 136.2    | 168.2           | 148.9         |
| Kuwait               | 62,664      | 130.2    | 160.8           | 142.3         |
| Australia            | 61,789      | 128.4    | 158.5           | 140.4         |
| Denmark              | 59,889      | 124.5    | 153.7           | 136.0         |
| Sweden               | 57,114      | 118.7    | 146.5           | 129.7         |
| Canada               | 50,344      | 104.6    | 129.2           | 114.4         |
| Netherlands          | 50,085      | 104.1    | 128.5           | 113.8         |
| Austria              | 49,581      | 103.1    | 127.2           | 112.6         |
| Finland              | 48,812      | 101.5    | 125.2           | 110.9         |
| United States        | 48,112      | 100.0    | 123.4           | 109.3         |
| Ireland              | 47,478      | 98.7     | 121.8           | 107.9         |
| Belgium              | 46,608      | 96.9     | 119.6           | 105.9         |
| Singapore            | 46,241      | 96.1     | 118.6           | 105.0         |
| Japan                | 45,903      | 95.4     | 117.8           | 104.3         |
| United Arab Emirates | 45,653      | 94.9     | 117.1           | 103.7         |
| Germany              | 44,021      | 91.5     | 112.9           | 100.0         |
| Iceland              | 43,967      | 91.4     | 112.8           | 99.9          |
| France               | 42,379      | 88.1     | 108.7           | 96.3          |
| United Kingdom       | 38,974      | 81.0     | 100.0           | 88.5          |

**Base weighting.** The most straightforward way of combining indices is to calculate a weighted average. If 20% of the budget goes on food and